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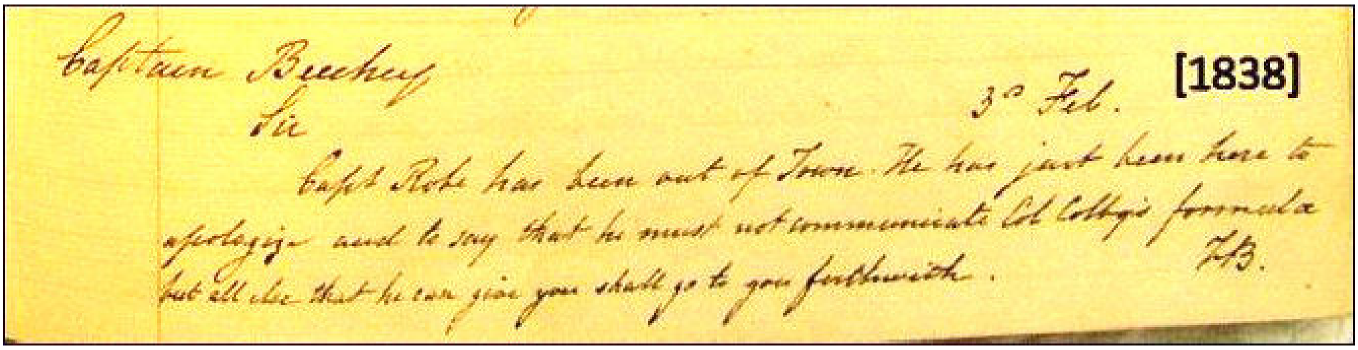
The Ordnance Survey and Airy's figure of the earth

David L Walker

The initial triangulation of Great Britain was marred by flawed estimates for the lengths of degrees that were used to calculate latitudes and longitudes.[1] For about thirty years, the Ordnance ignored criticism of its 1795 estimate of the length of a degree perpendicular to the meridian, as described below.

New estimates became available with the publication in 1830 of Airy's figure of the earth (ie its shape and dimensions) and these were fairly soon adopted by Ordnance officers. However, in 1838 Captain Robe RE had to apologise to the Admiralty's Hydrographer that "he must not communicate Col Colby's formula" for the length of a degree and ellipticity of the earth (*figure 1*). The relaxation of this prohibition in 1841 marked Colby's acceptance of 'Airy's spheroid', and this still provides the basis for the projection of Ordnance Survey maps.



Captain Beechey

3d Feb. [1838]

Sir

Capt Robe has been out of Town. He has just been here to apologize and to say that he must not communicate Col Colby's formula but all else that he can give you shall go to you forthwith.

F.B.

Figure 1: Letter from Capt F Beaufort, Admiralty Hydrographer, to Capt FW Beechey RN (UK Hydrographic Office (UKHO), LB 8, 93, Capt Beaufort to Capt Beechey, 3 Feb 1838)

The context

Isaac Newton postulated that the figure or shape of the earth was necessarily an oblate spheroid,[2] due to its adjustment to the effective reduction in gravity between the pole and the equator that arises from the increase in the offsetting centripetal force. In the first half of the early eighteenth century, this figure was sharply debated in the French Academy of Sciences until, after observations in Sweden and Peru, members agreed upon the model of an oblate spheroid.

In London the Astronomer-Royal in 1787 regarded "the true figure and dimensions of the Earth" as still uncertain.[3] However, William Roy in 1790 used a figure not very different from that adopted by the French, and concluded that it was 'sufficiently obvious that the earth … must be an oblate spheroid'.[4]

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[1] David L Walker, *A fresh look at the initial Ordnance triangulation etc.*, *Sheetlines* 117, 9-22.

[2] An oblate spheroid is an ellipse rotated on its minor axis. If rotated on its major axis, it is a prolate spheroid. All spheroids are ellipsoids but not all ellipsoids are spheroids.

[3] Sir Charles Close, *The early years of the Ordnance Survey,* 1926; republished with an introduction by JB Harley, Newton Abbot: David & Charles, 4, 1969.

[4] Maj.General William Roy, *An Account of the Trigonometrical Operation, whereby the Distance between the Meridians of the Royal Observatories of Greenwich and Paris has been determined*, Phil. Trans. Roy. Soc. London, vol. 80, 212, 1790.

After Roy's death, William Mudge effectively took charge of the Survey, with Isaac Dalby still providing mathematical advice. In 1795 (by which time Britain was at war with France), they published their determination of 61,182 fathoms for the length of a degree perpendicular to the meridian at latitude 50 degrees 41′ N.

The distance between the meridians of Dunnose and Beachy Head was calculated from the triangulation, which included observations extending up to 64 miles. This was divided by the difference in longitude calculated from the spheroidal triangle PDB, which was dependent upon only a few observations of the pole star.

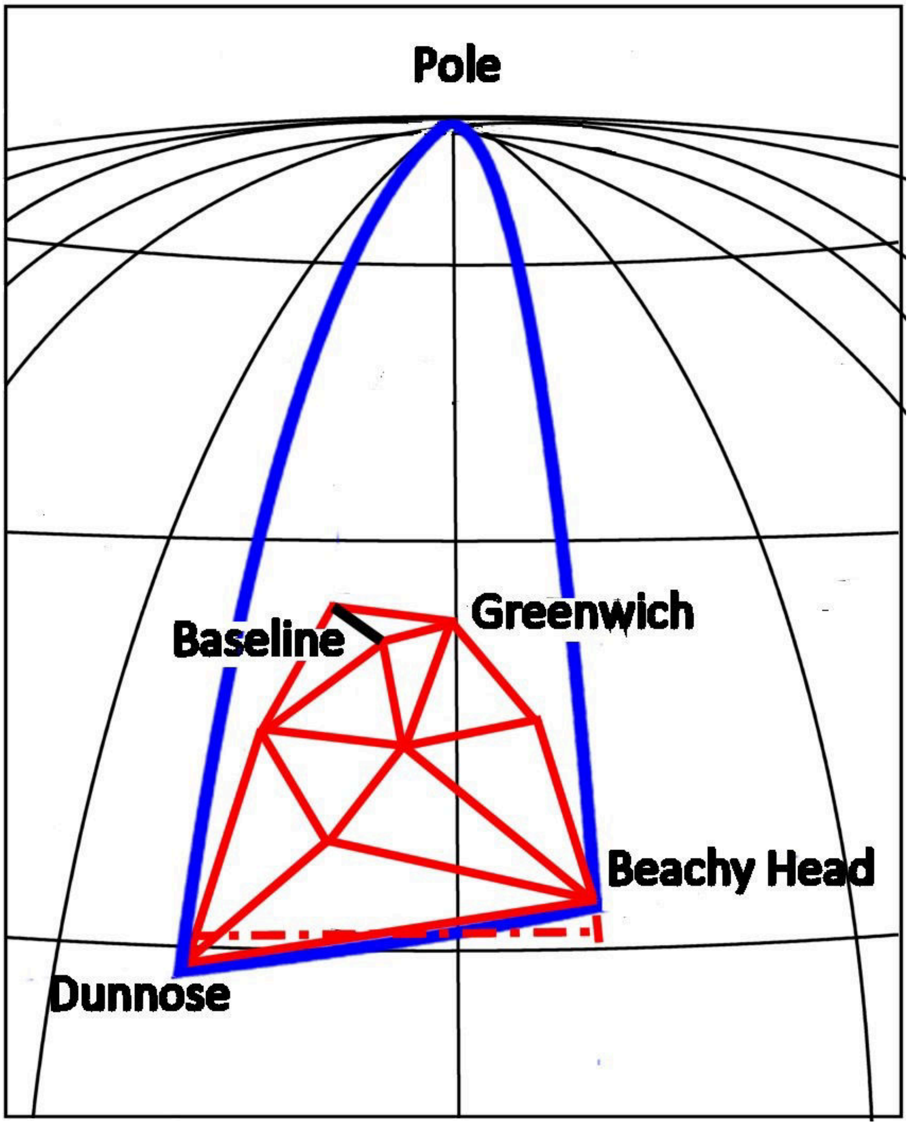

Figure 2 Determination in 1795 of length of a degree (exaggerated scale)

Yet Mudge and Dalby felt sufficiently confident to declare, in relation to foreign observations, that "This comparison … sufficiently proves that the earth is not an ellipsoid", and, five years later, for Mudge to declare that there was "great reason to suppose that the earth is not any regular figure".[5] Charles Close concluded that, in the absence [as they saw it] of an accepted figure of the earth, "in practice the officers of the Ordnance Survey, at the end of the eighteenth century, measured the curvature of the earth in two directions as they went along".[6] But it seems illogical that Mudge, having rejected a regular figure of the earth, adopted a constant length of a degree of longitude across the country.

Far-sighted criticism from Professor John Playfair in 1798

Although the improved assumptions eventually adopted by the Ordnance are known as 'Airy's spheroid', his approach was described thirty years earlier in a little recognized paper by John Playfair [7] that provides a useful introduction to this topic. Playfair's eclectic output benefited from his friendships with the revolutionary geologist James Hutton and the Astronomer-Royal, Neville Maskelyne, whom he had assisted at the measurement of Schiehallion in 1774.

The argument of Playfair's paper is remarkably clear, succinct and far-sighted. From the eighteenth century research (and his own planetary observations?), he firmly regarded the earth as an oblate spheroid. In his view, variations in the ellipticity (or eccentricity) of around 1 in 300 deduced by various observers resulted from irregularities in gravitational attraction on a plumb-line (or a spirit level) arising from variations in the density of underground strata as well as the surface terrain. From his geological knowledge, he calculated that these variations could distort a plumb line by 10 or 12 seconds. As they were unavoidable, he concluded that errors in observing latitudes could be contained only by confining comparisons to arcs of at least several degrees.

Playfair nicely demonstrated the equation for the observed length L of an arc of an elliptical meridian between two observed latitudes λ and λ' in terms of the earth's equatorial semi-axis 'a' (or its polar semi-axis 'b') and its eccentricity 'e' (defined as (a-b)/b). Another arc observed on any meridian (preferably at a very different latitude) could provide a second equation. Hence a and e (or b and e) could be determined from two equations containing two unknowns. This was also the basis for the approaches to the figure of the earth adopted by Lambton in 1818 and Airy in 1830.[8]

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[5] Lt Col Edward Williams, Capt William Mudge RA and Mr Isaac Dalby, *An Account of the Trigonometrical Survey carried on in the years 1791, 1792, 1793, and 1794,* Phil. Trans. Roy. Soc. Lond., vol 85, 511-527, 1795; Capt William Mudge RA, *An Account of the Trigonometrical Survey carried on in the years 1797, 1798, and 1799 etc*, Phil. Trans. Roy. Soc. Lond., vol 90, 636, 1800.

[6] Sir Charles Close, *The early years etc.,* 149, republished 1969.

[7] John Playfair, Professor of Mathematics in the University of Edinburgh*, Investigation of certain theorems relating to the figure of the earth [read 5 Feb 1798]*, Trans. Roy. Soc. Edinburgh, Vol V, 1805. https://archive.org/details/transactionsofro05murr/page/n11/mode/2up

[8] The equation for the length of an arc, using Airy's notation, is shown in the Appendix.

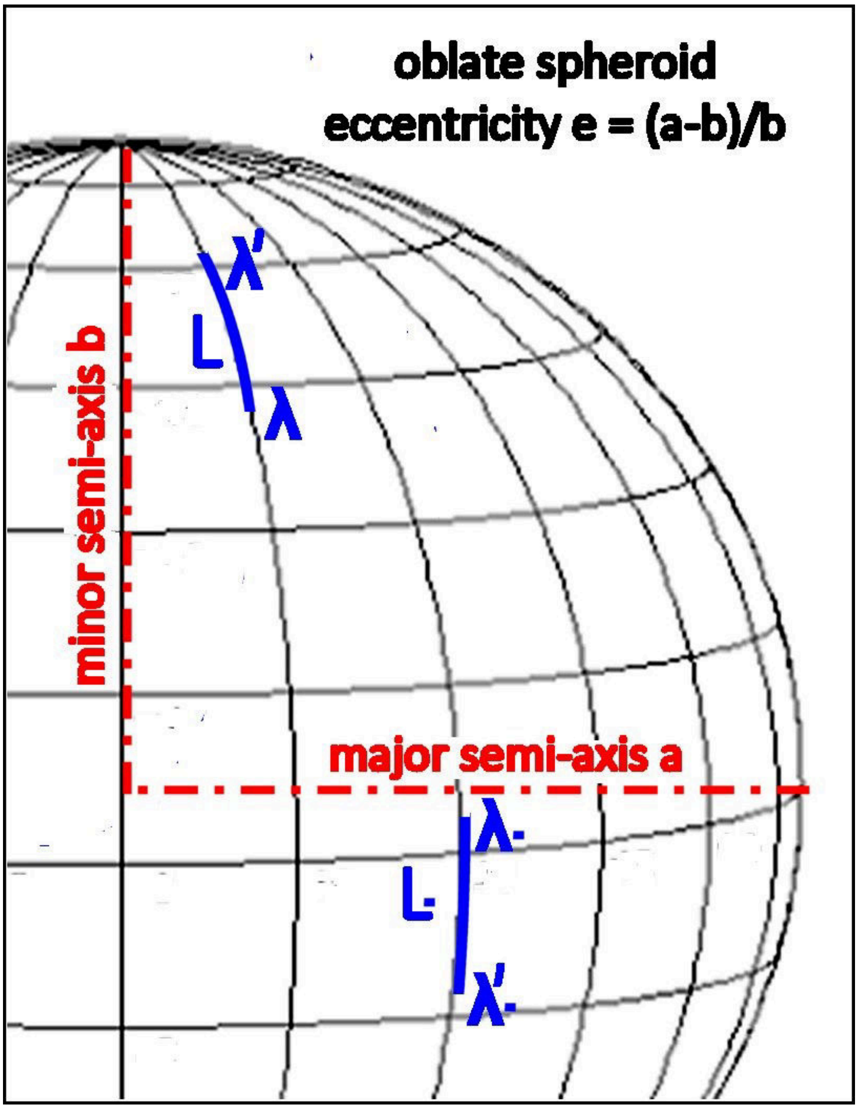

Figure 3 : Approach to determining the figure of the earth from two observed arcs of the meridian

Playfair found it "not easy to account for" the ellipticity of the earth implicit in the 1795 paper by Mudge and Dalby. He did not question their trigonometry, but, significantly, warned that, if the difference in longitude observed between Beachy Head and Dunnose had been affected by the reduced gravitational attraction of the lighter underground strata between them, other longitudes derived from these observations "[would] appear less than they ought to do".

Without any further validation, the Ordnance somewhat rashly used the length of a degree of longitude estimated in 1795 to determine the longitudes of the various meridians adopted for the initial triangulation, as it reported from 1795 until 1811. In 1813, Mudge advised Colby that "Mr Playfair is a man of great natural sagacity and much acquired information",[9] but unfortunately they had paid more attention to Playfair's compliments than to his tactful criticism.

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[9] Mudge to Colby, 9 Sept 1813, quoted in Sir Charles Close, *The early years etc.,* 59, 1969.

Figure of the earth published by William Lambton in 1818

Lambton's talent for mathematics and his surveying experience enabled him in 1800 to take the opportunity, when serving under Colonel Arthur Wellesley, to commence the great triangulation of India that he led until his death in 1823. Under-rated in London, it was not until Lambton was honoured by the French Academy of Sciences in 1817 that he was elected a Fellow of the Royal Society.

Lambton's triangulation was published in this country in 1818.[10] As it was at a low latitude, it provided a well-placed arc of the meridian. From his own triangulation, taken with that by Legendre in France, he firmly deduced a spheroidal figure of the earth. This was tabulated very usefully in terms of the lengths of degrees on and perpendicular to a meridian, at three-degree intervals from the equator to the pole. By reference to later figures of the earth, Lambton's estimates proved more accurate than previous figures.

A general defect in the Ordnance longitudes found by John Tiarks in 1824

On the recommendation of Sir Joseph Banks, John Lewis Tiarks became British Astronomer to the Boundary Commission for the border between the United States and Canada. From there, he was appointed by the Admiralty to measure the longitude of Falmouth, first from Greenwich in 1822 using 15 chronometers, and then from Dover in 1823 using 26 chronometers.

His report to the Board of Longitude, published by the Royal Society,[11] boldly concluded that "it is a general and proportionate defect of all longitudes deduced from the [Ordnance Trigonometrical] Survey, and not the erroneous longitude of any particular station, which has caused the disagreement between the results of the chronometers and of the Survey." Supposing his final result to be correct, "all the longitudes in the Account of the Survey must therefore be increased" in the proportion 4.92" to 25' 23.5" [of time]. It is a tribute to his accuracy that this is exactly the same as the increase of 11.6 seconds per degree that was later adopted by the Ordnance.[12]

Tiarks also was sceptical of any determination of the length of degree perpendicular to the meridian by geodetical measurement "independently of any hypothesis respecting the figure of the earth." If the Ordnance figure was correct, he argued, the ellipticity of that part of England must be very different from the earth as a whole and therefore unreliable for the country as a whole. As degrees of latitude could be determined more accurately than degrees of longitude, it was preferable to determine the figure of the earth from two well-separated arcs of latitude, and to use this figure to calculate the length of a degree of longitude.

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[10] Lt Col William Lambton, 33rd Regiment of Foot, *An abstract of the results deduced from the measurement of an arc on the meridian … being an amplitude of 9° 53' 45.2",* Phil. Trans. Roy. Soc. London, vol. 108, 486-517, 1818.

[11] Dr John Lewis Tiarks, *A Short Account of Some Observations Made with Chronometers, in Two Expeditions Sent Out by the Admiralty, at the Recommendation of the Board of Longitude, for Ascertaining the Longitude of Madeira and of Falmouth,* Phil. Trans. Roy. Soc. London, vol. 114, 360–371, 1824.

[12] WA Seymour (ed), *A history of the Ordnance Survey*, 39, Dawson, 1980.

Doubts published by Captain Kater in 1828

Having assisted Lambton on the trigonometrical survey of India, Henry Kater retired from the army in 1814 and enjoyed an independent scientific career over the next twenty years. As the Royal Society nominee on the re-triangulation between Paris and London, he enjoyed considerable independence but relied on the support of the Ordnance surveyor James Gardner.

Unlike the Ordnance, Kater assumed that the earth was a spheroid with an eccentricity of about 1 in 300. He found the length of a degree perpendicular to the meridian significantly less than the Ordnance figure (and much closer to later figures) but rejected his own estimate. Instead, after discussing the unavoidable inaccuracies in making observations of the pole star, he concluded that such observations, “for the purpose of determining the length of the perpendicular degree in our latitude, are wholly unworthy of credit …” [13]

The brief adoption by the Ordnance of Lambton’s figure of the earth

After ‘the three volumes’ of the initial triangulation were published in 1811, Thomas Colby, who was promoted to Superintendent of the Survey in 1820, proceeded with the triangulation of Scotland until he was diverted to the Irish survey from 1823. However, no survey results were published by the Ordnance after 1811, although they supplied a few latitudes and longitudes to the Admiralty.

In 1812, Don Rodriquez had strongly criticized the uneven estimates by the Ordnance of lengths of degrees on the Dunnose meridian. After his criticisms were disputed by other geodesists, this issue seems to have come to rest. However, by 1828 no fewer than four Fellows of the Royal Society, as mentioned above, had separately expressed doubts over the determination by the Ordnance of the length of a degree perpendicular to the meridian. There is no sign in the various histories that Colby responded to these doubts. Nevertheless, the Hydrographer’s files show that Lt Hastings Murphy of the Ordnance, with Captain Robe’s knowledge, did recalculate some latitudes and longitudes for the Admiralty. In April 1830, he provided Lt Slater RN with positions for five stations on the Northumberland coast and three stations near the Scottish border. These three figures differed by about 10 seconds (south) in latitude and 30 seconds (west) in longitude from those published in 1811. Another letter from Murphy demonstrates that these figures were calculated “on the spheroid of Lambton & Delambre” (as published in 1818).

Lambton’s figure of the earth was also adopted by Lt Denham RN in April 1830 in making a comparison for four stations in Pembrokeshire between the latitudes and longitudes published in 1811 and his own figures “as recomputed on the spheroid of Delambre & Lambton”. [14]

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[13] Capt Henry Kater, *An account of trigonometrical operations in the years 1821, 1822 and 1823, for determining the difference of longitude between the Royal Observatories of Paris and Greenwich*, Phil. Trans. Roy. Soc. London, vol. 118, 188-189, 1828.

[14] David L Walker, *A fresh look etc., Sheetlines* 117, 20, 2020 provides a fuller account of the Don Rodriquez, Murphy and Denham episodes, with references.

Professor George Biddell Airy 1801-1891

Having regard to the enduring use of Airy's spheroid, it is surprising to find that in 1830 it was published not in an academic journal but as an article in the little-known *Encyclopaedia Metropolitana*.[15] Planned by the poet Samuel Taylor Coleridge to provide a systematic approach to all knowledge, this ran to thirty volumes but failed to compete with the *Encyclopaedia Britannica*. As well as the figure of the earth, Airy provided articles on trigonometry, waves and tides.

In 1824, at the age of 23, the impecunious Airy had a proposal of marriage refused by the lady's father. The fee of £100 for the figure of the earth was worth half his initial stipend at Cambridge, and even after securing the well-remunerated Plumian chair in astronomy his autobiography reveals continuing financial concerns. His recollection that in November 1829 he "wrote some (perhaps much)" of his article is followed by a brief note of the acceptance of his marriage proposal.[16] That he specifically recalled receiving his £100 fee in 1831 suggests that this motivated the publication of his figure of the earth in an encyclopaedia.

Airy's article on the figure of the earth

Although Airy's autobiography reveals little evidence of field experience in trigonometrical surveying, his article demonstrates a mastery of the subject gained from his extensive reading, useful site visits and voluminous overseas correspondence. Moreover, his article was not confined to determining a figure of the earth from geodetical measurements. He analysed the theory and practice of pendulum observations to deduce the shape of the earth from the variation in gravity between the pole and the equator. He also considered its determination from his understanding of the earth's precession and nutation, and used these alternative methods to support but not to modify his geodetic conclusions.

In his article of 76 pages, the geodetical analysis occupied only 6 pages and does not involve much more than sixth form mathematics. His approach was much the same as that outlined thirty years earlier by John Player, and endorsed by Lambton and Tiarks. However, Airy could now draw upon fourteen measurements of arcs of the meridian (*figure 4*), although his article does not seem to express any thanks for this hard-won body of work.

A notable feature of Airy's mathematics, as described in the Appendix to this article, is that he firmly rejected an apparently objective least squares analysis of the data. Given that any pair of arcs was sufficient to calculate values of the earth's diameter and its ellipticity, he simply examined the values calculated from selected pairs of arcs or groups of arcs. After also applying his opinions of their accuracy and of gravitational disturbance, he discarded some arcs, used others more than once and finally averaged the results from two groups of arcs.

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[15] GB Airy, *Figure of the Earth*, Encyclopaedia Metropolitana, vol V, 165-239, revd. 1848.
[16] Wilfred Airy ed., *Autobiography of Sir George Biddell Airy*, Camb. Univ. Press, 89, 1896.

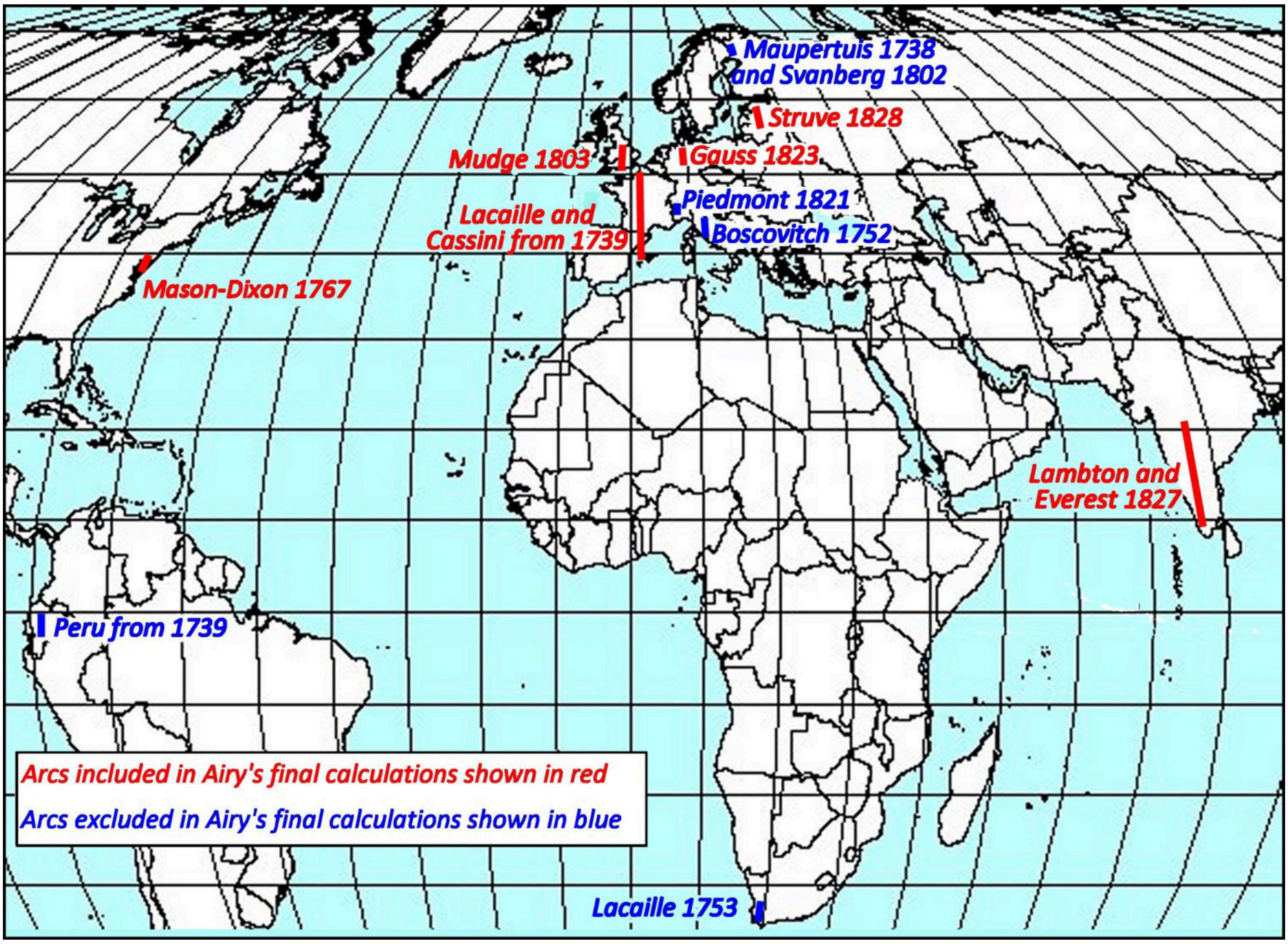

Figure 4 : Measured arcs referred to in Airy's paper
(and listed in the Appendix to this article)

The hesitant adoption by the Ordnance of Airy's figure of the earth

Ordnance papers, scarce before 1840, do not appear to record when Airy's figure of the earth was adopted for calculations of latitude and longitude. His article was dated 17 August 1830 and there is a note in TNA 2/616 [17] that "Stations in this volume said to be on Airy's figure are computed from elements supplied by the late Professor Airy in the year 1830-31." But, as Airy died in 1891, this note must have been written many years later.

The earliest clue found in the Hydrographer's letter books is Murphy's letter of 6 December 1834 that provided revised positions in eastern Scotland that can be shown to be based on Airy's figure of the earth.[18] On the other side of the country, Captain Alexander Henderson RE was working independently on his impressive triangulation of the coasts of the Irish Sea.[19] Although his calculation

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[17] TNA, OS 2/616, *Latitudes and longitudes of the Principal Stations on Airy's 1830 Figure and the British Figure of 1858,* 1856-1867.

[18] David L Walker, *A fresh look etc., Sheetlines* 117, 20, 2020 describes Murphy's advice.

[19] David L Walker and Adrian Webb, *Some collaboration between the Ordnance Survey and the Hydrographic Office etc.*, *Sheetlines* 102, 10-13, 2015; UKHO, L2944, Criffell 54° 56' 28" N, 3° 37' 37.2" W, 1836; L2944a, Bengairn, 54° 52' 14.6" N, 3° 54' 56.7" W, 1836.

books have not survived, his triangulation diagrams show latitudes and longitudes for Criffell and Bengairn that definitely were recorded in 1836 and can be shown to be calculated on Airy's figure of the earth.

Captain Beechey, now responsible for the Admiralty survey of the Irish Sea, had worked in the Pacific and clearly understood the figure of the earth. Unhappy about differences between latitudes and longitudes supplied by Henderson and those published in the 'three volumes', Beechey sought a quantified explanation from the Ordnance. This in 1838 provoked the strange response from Captain Robe, referred to at the start of this article, that "he must not communicate Col Colby's formula" for the length of a degree and ellipticity of the earth.

So why was Colby unwilling to explain that these differences arose from the adoption of Airy's figure of the earth? Was he hesitant over abandoning figures inherited from Mudge? Or, having devoted more than ten years to the survey of Ireland, was he simply out of touch? The reader must decide.

However, Colby was ready to accept advice from Airy, by then the Astronomer Royal, on the calculation of latitudes and longitudes using polar co-ordinates taken directly from trigonometrical observations (instead of calculating latitudes and longitudes from Ordnance 'Cassini' co-ordinates).

As a test of this method (not of Airy's spheroid), Captain Portlock used data from the Scottish triangulation to calculate the latitudes and longitudes of a closed chain of a dozen stations.[20]

Soon after this, in a letter dated 15 June 1841, Yolland provided the lengths of degrees that Robe had been forbidden to provide in 1838.[21] Thus it seems that Colby by 1841 had accepted that the Ordnance should adopt Airy's figure of the earth.

The survival of Airy's figure of the earth

Despite international improvements to the figure of the earth, 'Airy's spheroid' is still, after nearly two centuries, recognised on current Ordnance sheets as the basis for the National Grid (and for sheetlines). So how has it survived for so long?

The first potential change came in 1858 when Captain Alexander Ross Clarke RE calculated a figure of the earth that provided a best fit to British observations, and slightly revised latitudes and longitudes for the Ordnance stations using this 'British figure'.[22] These were listed, followed by a longer list of latitudes and

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[20] David L Walker, *Troubled progress of the Scottish triangulation etc., Sheetlines* 104, 11-12, 2015.

[21] UKHO, LP 1857 Y, 15 June 1841. The lengths of degrees match those tabulated for Airy's spheroid in Yolland, *Measurement of the Lough Foyle base etc.,* 115, Ordnance Survey, 1847.

[22] Capt Alexander Ross Clarke under the direction of Lt Col H James, *Account of the Observations and calculations of the Principal Triangulation and of the Figure, Dimensions and Mean Specific Gravity of the Earth as derived therefrom,* Ordnance Survey, 1858.

longitudes on Airy's figure, in the Ordnance file mentioned above. But an unsigned note inside the cover decrees "Airy's Figure only to be used". [23]

By then the Ordnance was showing a marginal scale of latitudes and longitudes on its large-scale plans, calculated on a sheet by sheet basis using Airy's figure of the earth. As there was financial pressure from the Treasury, as usual, perhaps it was decided that the cost of revising Ordnance tables and recalculating these scales on Clarke's spheroid would exceed the potential benefits.

Reflecting on this policy much later, Charles Close noted that a change to the spheroid favoured at that time (1926) would alter the latitude of Cape Wrath by about 3½ seconds and he endorsed the view that "no practical or theoretical purpose would be served by re-computing the geographical positions." [24]

The procedure adopted in 1936 for the re-triangulation of Great Britain was to re-observe a central network of eleven former stations, and then to adjust the position, scale and direction of the combined figure "so as to give a best mean [least squares] fit with the Principal Triangulation co-ordinates at the eleven points, the shape of the combined figure remaining undisturbed." As a consequence, Airy's figure of the earth remained as the basis for the projection of Ordnance maps, now using the Transverse Mercator Projection. Thus 'Airy's spheroid' also became the basis for the National Grid (and for sheetlines).

A more recent justification for the retention of Airy's figure is that "The ellipsoid used for mapping in Britain, the Airy 1830 ellipsoid, is designed to best-fit Britain only, which it does better than GRS80 (Geodetic Reference System 1980), but it is not useful in other parts of the world." [25] It is difficult to reconcile this 'best-fit' justification with Airy's conclusion (described in the Appendix).

Conclusions

Confident in the quality of their instruments, their observations and their calculations, Superintendents of the Survey were slow to recognize the effect of gravitational variations attributable to underground strata as well as hills and mountains. If they had paid more attention to outsiders, they might sooner have appreciated these crucial influences on the plumb line and the spirit level.

Nevertheless, the Ordnance had the wisdom, however cautiously, to adopt the figure of the earth published by Professor Airy in 1830. As unnecessary changes to this 'near enough' figure were avoided over subsequent years, the role of the 'Airy spheroid' is still recognised on every Ordnance sheet.

Acknowledgement

For encouragement and advice, the writer thanks Rob Wheeler; Jim Smith, author of '*From Plane to Spheroid*', 1986; and Adrian Webb, Archive Services Manager at UKHO; and UKHO is thanked for permission to reproduce *figure 1*.

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[23] TNA, OS 2/616, *Latitudes and longitudes of the Principal Stations on Airy's 1830 Figure and the British Figure of 1858,* 1856-1867.

[24] Sir Charles Close, *The early years of the Ordnance Survey etc.*, 149, republished 1969.

[25] *A guide to coordinate systems in Great Britain*, Ordnance Survey, 6, 2015.

Appendix : Airy's method for determining the figure of the earth[26]

1. Airy assumed the earth's surface to be an oblate spheroid, with a major (equatorial) semi-axis = **a** (in feet), minor (polar) semi-axis = **b** (in feet) and ellipticity **e = (a-b)/b** (as in *Figure 3*). For a spheroidal earth, meridians of longitude form ellipses and the latitude λ of a point is defined as the angle between the perpendicular through that point (not the radius) and the equatorial axis.

2. Using symbols as above (and supposing **e** to be small enough to neglect $\mathbf{e}^2$), Airy demonstrated:

radius of a parallel = $\mathbf{b} \cos\lambda\,(1 + \mathbf{e} + \mathbf{e}\sin^2\lambda)$ (page 193)

radius of curvature of a meridian = $\mathbf{b}\,(1 - \mathbf{e} + 3\,\mathbf{e}\sin^2\lambda)$ (page 194)

3. Using this radius of curvature, Airy (like Playfair and Lambton) integrated elements of an elliptical meridian from latitude λ to latitude λ′to define the length of an arc of the meridian $L = \mathbf{b}\,((1 + 0.5\,\mathbf{e})(\lambda' - \lambda) - 1.5\,\mathbf{e}\cos(\lambda' + \lambda)\sin(\lambda' - \lambda))$ (page 194). Using this formula for the length of an arc of the meridian, he formed 'equations of condition' from the observed latitudes and lengths of each of the 14 arcs shown in *figure 4*, thereby forming 14 equations from the data in Table 1.

	Airy 1830 Location	Date approx	Ascribed to	Latitude mid point	Amplitude	Length English feet	Kater's remarks (some abbreviated)	Final weighting /48
Arcs of Meridian								
1	Peru	1751	Bouguer etc	-1°31'00"	3°07'03.1"	1,131,057	Mountainous. Amplitude probably good.	-
2	Sweden	1739	Maupertuis	66°19'37"	57'30.4"	351,832	Mountainous. A little doubt re amplitude	-
3	French 1	1798	Lacaille etc	46°52'02"	8°20'00.3"	3,040,605	Apparently very good.	2
4	Roman	1752	Boscovitch	42°59'00"	2°09'47.0"	787,919	Mountains mid arc. Determination good.	-
5	The Cape	1753	Lacaille	-33°18'30"	1°13'17.5"	445,506	Mountainous. Determination good.	-
6	America	1767	Mason Dixon	39°12'00"	1°28'45.0"	538,100	Favourable country: apparently good.	2
7	French 2	1806	Biot Arago	44°51'02"	12°22'12.6"	4,509,402	Latitudes: inadequate observations.	5
8	Sweden	1802	Svanberg	66°20'10"	1°37'19.3"	593,278	Mountainous. Latitudes doubtful.	-
9	English	1803	Mudge	52°35'45"	3°57'13.1"	1,442,953	Excellent.	5
10	Indian 1	1823	Lambton	12°32'21"	1°34'56.4"	574,368	Excellent.	12
11	Indian 2	1829	L'ton Everest	16°08'22"	15°57'40.2"	5,794,599	Excellent.	12
12	Piedmont	1821	Plana etc	44°57'30"	1°07'31.1"	414,657	Determination excellent: mountainous.	-
13	Hanover	1823	Gauss	52°32'17"	2°00'57.4"	736,426	Excellent but doubt re standard.	5
14	Russian	1822	Struve	58°17'37"	3°35'05.2"	1,309,742	Excellent.	5
Arcs of Parallel								
15	Rhone	1740	Lacaille etc	43°31'50"	1°53'19.0"	503,022	Pretty good.	-
16	Dunnose	1795	Mudge	50°44'24"	1°26'47.9"	336,099	Apparently very good.	-
17	Falmouth	1824	Tiarks	50°44'24"	6°22'06.0"	1,474,775	Apparently good.	-
18	Padua	1824	Beccaria etc	45°43'12"	12°59'03.8"	3,316,976	Accumulated errors in circumstances.	-

Table 1 Measured arcs considered by Professor Airy in 1830

[26] GB Airy, *Figure of the Earth*, Encyclopaedia Metropolitana, vol V, 165-239, revd. 1848.

4. For example, using ($\lambda' - \lambda$), measured in radians, the reader may verify the 'equation of condition' for arc no.9 as 0.0690040**b** + 0.06160**be** = 1442953 and using eg arc no 10 may calculate a figure of the earth from these two equations.

5. To combine his 14 equations, Airy firmly rejected the method of least squares (although this was developed by Legendre, Gauss and later Bessel specifically for their analyses of the figure of the earth). Instead Airy formed simultaneous equations between selected pairs of these equations (or the sums of several equations), and applied his own judgement to the variations between the results. Having chosen and solved nine equations, he discarded six meridians that in his view gave discordant results, for reasons that he explained (mostly by reference to nearby mountains).

6. Next he 'thought best to use the method commonly employed in astronomy viz. to take the sum of groups of the equations of condition, and to consider each sum as one equation: the groups being selected so as to make the coefficient of e large and positive in one sum, and large and negative in the other.' On this basis, he finally gave equal weighting to two simultaneous equations:
(a) sum of six 'higher latitude' equations v sum of two Indian equations; and
(b) sum of the recent four out of above six v sum of two Indian equations.
The chosen arcs and their effective weighting are shown in Table 1.

7. Hence, he concluded, "[the] measured arcs may be represented nearly enough *on the whole* by supposing the Earth's surface (at the level of the sea …) to be an ellipsoid of revolution, whose polar semi-axis is 20,853,810 English feet … and whose equatorial semi-axis is 20,923,713 feet …" (and with ellipticity 1:298.33).

8. Airy also formed equations of condition from the observations of the four arcs of parallel shown in Table 1. Having described some of these rather uncritically as good, he decided from the results that arcs 15 and 16 had been disturbed by the attraction of hills at both extremities. He regarded the results from arcs 17 and 18 as compatible with his results from arcs of meridian.

9. Thus Airy's well-regarded and lasting work used relatively simple mathematics. Although Charles Close described his technique as 'arbitrary',[27] Airy's subjective decisions allowed a more thoughtful approach than least-squares analysis.

10. In 1837-41, Bessel applied least-squares analysis to fit a meridian arc to 38 points from a similar set of measured arcs and his results were little different from Airy's.[28] Apparently Bessel's figure is still used in some parts of Europe where, as for Airy's figure in Britain, the benefits of a change have been judged not worthwhile compared with the costs.

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[27] Sir Charles Close, *The early years of the Ordnance Survey etc.*, 148, republished 1969.

[28] Airy is closer to the modern figure of the earth for **a** and **b**, and Bessel for **e.**