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“A fresh look at the initial triangulation”

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The Charles Close Society was founded in 1980 to bring together all those with an interest in the maps and history of the Ordnance Survey of Great Britain and its counterparts in the island of Ireland. The Society takes its name from Colonel Sir Charles Arden-Close, OS Director General from 1911 to 1922, and initiator of many of the maps now sought after by collectors.

The Society publishes a wide range of books and booklets on historic OS map series and its journal, *Sheetlines*, is recognised internationally for its specialist articles on Ordnance Survey-related topics.

A fresh look at the initial Ordnance triangulation of 1795-1811

David L Walker

The series of papers published on behalf of the Ordnance from 1795 until 1811 recorded the triangulation of Great Britain (including only southern Scotland) in impressive detail.¹ These papers, accessible on the Royal Society website, provide very readable accounts of the surveyors' journeys, their observations and their calculations.

Although the triangulation was observed as an interconnected network, its vertices or stations were generally related to one of nine chains of triangles, each referred to the meridian of a station designated as an origin (*figure 1*). The siting of stations, the observations made from them, and the calculation from the Ordnance baselines of the sides of the triangles, are well described in the papers.

However, there is less explanation of the outputs from this work, that is the calculations of the position of each station in terms of co-ordinates 'from the parallels and perpendiculars to their respective meridians' for use by topographic surveyors (*table 1*), and the conversion of these co-ordinates into latitudes and longitudes for use by hydrographic surveyors (*table 2*).

Table 1
Co-ordinates

(Extract from
Art. XXV,
Royal Society
1800, 643)

ART. XXV. Bearings of the Stations in the Series of 1795 and 1796, from the Parallels to the Meridians of Black Down, Butterson Hill, and St. Agnes Beacon; likewise their Distances from those Meridians, and from their Perpendiculars.					
<i>Meridian of St. Agnes Beacon.</i>					
Hensbarrow	-	St. Agnes Beacon	73 4 13 NE	92877,4	28279,9
Deadman	-	-	72 24 27 SE	97292,5	30849
Karnbonellis	-	-	3 27 27 SW	2741,7	45379,2
Karnminnis	-	-	61 13 58 SW	74168,1	40719
Bodmin	}	Hensbarrow	37 30 45 NE	121703,2	65825,8
Lansallos			75 29 51 SE	152945,3	12733,5
St. Burian	-	Karnbonellis	67 20 59 SW	94831,5	83807,3
Pertinney	-	Karnminnis	39 25 32 SW	100465,1	72704,4
Sennen	-	Pertinney	40 50 18 SW	113674,4	879868

Table 2
Latitudes and
Longitudes

(Extract from
Art. XXVI,
Royal Society
1800, 644)

ART. XXVI. Latitudes and Longitudes of the Stations in the Series of 1795 and 1796.				
<i>Meridian of St. Agnes.</i>	<i>Latitude.</i>	<i>From merid. of St. Agnes.</i>	<i>Longitude west of Greenwich</i>	
			<i>In degrees.</i>	<i>In time.</i>
Lansallos	50 20 25,7	0 39 10,3 E	4 32 45,7	18 11,0
Bodmin Down	50 29 11,6	0 31 15,9 E	4 40 39,8	18 42,6
Deadman	50 13 20,0	0 24 51,3 E	4 47 4,4	19 8,3
Karnbonellis	50 10 59,4	0 0 42,0 W	5 12 37,7	20 50,5
Karnminnis	50 11 43,8	0 18 56,2 W	5 30 51,9	22 3,5
St. Burian	50 4 37,9	0 24 9,2 W	5 36 4,9	22 24,3
Pertinney	50 6 27,0	0 25 36,2 W	5 37 31,9	22 30,1
Sennen	50 3 55,6	0 28 56,7 W	5 40 52,4	22 43,5

¹ These papers, published by the Royal Society in 1795, 1797, 1800 and 1803 and by William Faden in 1811, are referred to below by years of publication, with details listed in the Appendix.

The Royal Society papers may be accessed at <https://royalsocietypublishing.org/journal>

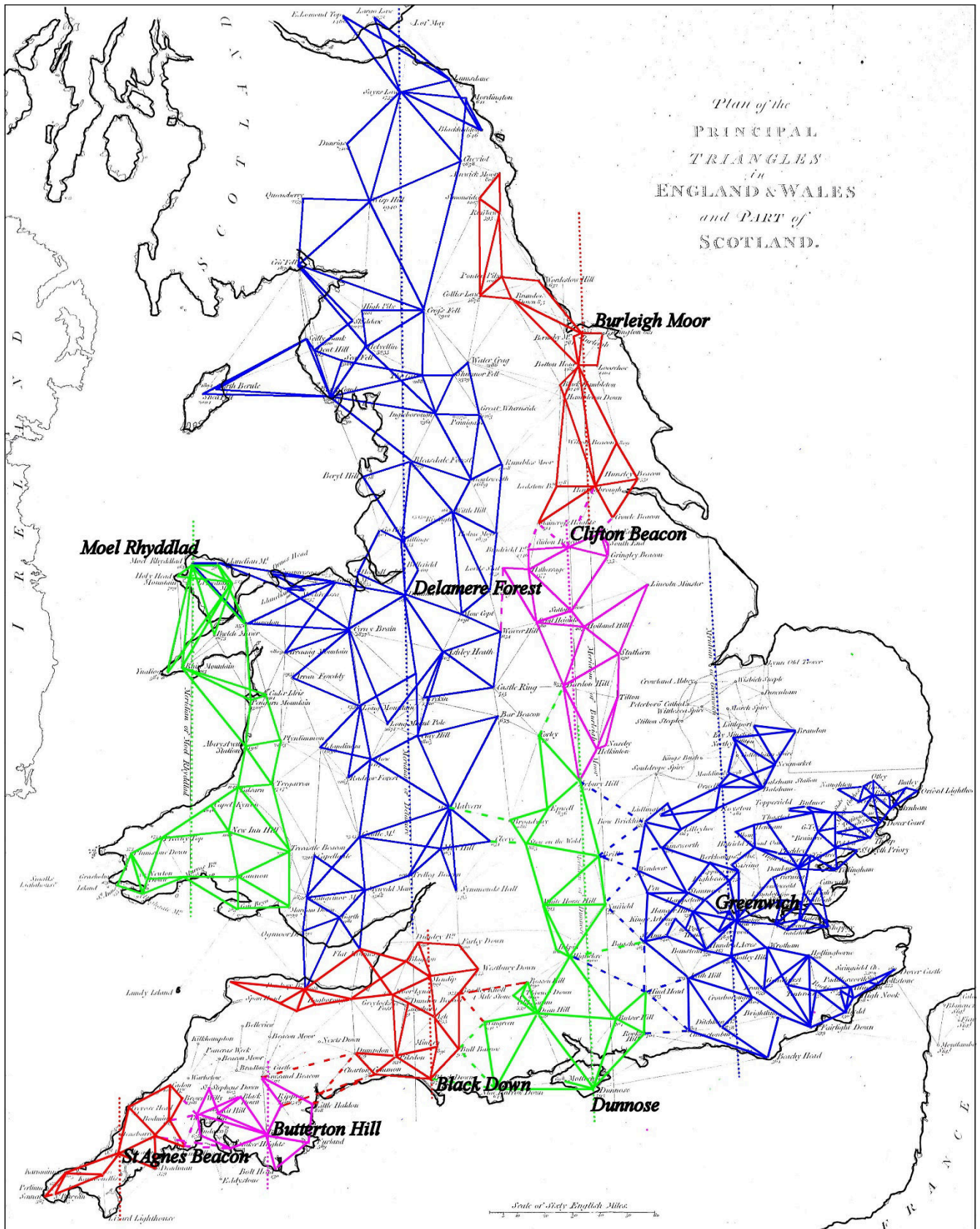


Figure 1: Ordnance stations related to meridians used for the initial triangulation (based upon plate I of vol III of *An Account of the Trigonometrical Survey*, published by Faden, 1811)

The considerable work of the late Brian Adams and others has shown that the use by the Ordnance of co-ordinates (as in table 1) to project the sheet-lines of their early maps was in several respects inconsistent.² This article is concerned with previous stages of this process, concerned with calculating the co-ordinates of each station and their conversion into latitudes and longitudes.

The calculation of station co-ordinates

‘Great triangles’, normally observed from a primary station at each vertex, are shown in figure 1. ‘Intersected stations’ observed from only two vertices were known as secondary stations. Triangles were observed as if spherical triangles, but calculated as plane triangles. Hence the calculated sides of these triangles were chords, not arcs, of the spheroidal earth (and calculated as if on a sphere).

The co-ordinates of each station were measured in feet (a) on the great circle passing through the station that was perpendicular to the meridian passing through its related origin, between station and meridian; and (b) on this meridian, between the point of intersection of the great circle and the origin.³ It has been pointed out that this projection, initially known as ‘projection by rectangular spheroidal co-ordinates’,⁴ corresponded with the ‘Cassini projection’ used by the Ordnance in later years.⁵ For those of us who find it difficult to visualise Cassini’s projection, a chocolate orange on its side provides a useful aide memoire:

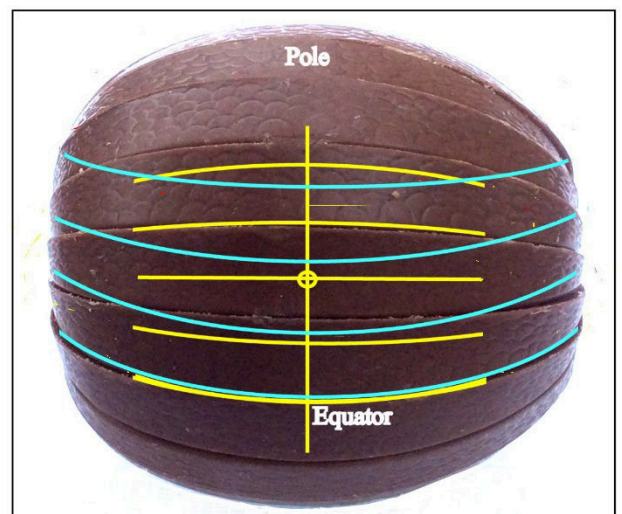


Figure 2 : Illustration of Cassini's projection

Chocolate segments show great circles of the earth (those on a plane through the earth's centre).

*Parallels of latitude are shown in light blue.
Cassini axes are shown in yellow.*

² Roger Hellyer and Chris Higley (eds), *Projections and Origins, collected writings of Brian Adams*, London, The Charles Close Society, 2006.

³ The inaccuracy arising from the difference between the length of an arc and the length of its chord was limited by aggregating the length of the co-ordinates step by step from one vertex to the next. For sides of (typically) 100,000 feet the difference between arc and chord is little more than an inch. But for longer sides the difference increases as the cube of the length.

⁴ Roger Hellyer and Chris Higley (eds), *Projections and Origins [etc]*, 22, The Charles Close Society, 2006.

⁵ RC Wheeler, ‘William Mudge and the General map of England’, *Sheetlines* 97, 18. This article considered several of the issues discussed here and stimulated these supplementary investigations.

Co-ordinates were calculated as follows, taking two stations measured from the origin of St Agnes Beacon as an example. *Figure 3* exaggerates the scale of the triangles compared with the earth to clarify observed chords (solid lines) and co-ordinates of these stations (dotted) on a great circle grid.⁶

(i) The meridian at St Agnes was observed from elongations of the pole star at morning and evening, and the observed bearing of Karnbonellis (K) from St Agnes was measured by reference to this meridian.

(ii) The distance of K from St Agnes was taken from the calculation of this great triangle. From bearing (i) and distance (ii), calculated as a plane triangle, the co-ordinates of K from St Agnes were measured as if on the meridian of St Agnes and on the great circle perpendicular to it.

(iii) At K, the bearing of St Burian (by reference to the meridian of St Agnes, not the meridian of K) was determined from the angle observed at K between St Agnes and St Burian.

(iv) The distance of St Burian from K was taken from the calculation of this great triangle. From bearing (iii) and distance (iv), calculated as a plane triangle, the co-ordinates of St Burian from K were measured as if on the meridian of St Agnes and on the great circle perpendicular to it.

(v) Co-ordinates (ii) were added to co-ordinates (iv) to determine the co-ordinates of St Burian from the origin (St Agnes).

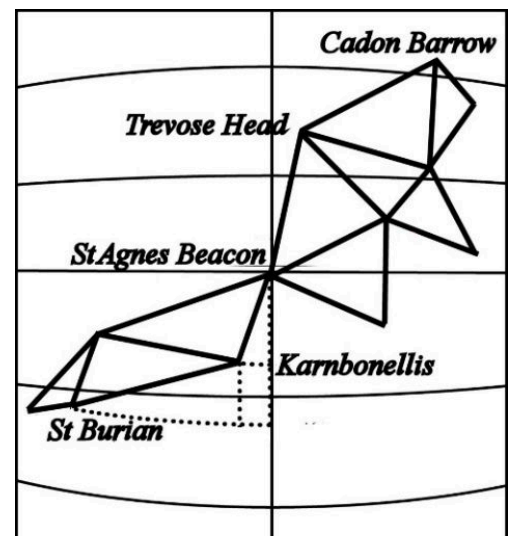


Fig 3 : Triangles measured from St Agnes Beacon

The bearing used to calculate each co-ordinate was shown in the list of co-ordinates (as in table 1) but the distance had to be found from the calculations of the sides of triangles shown previously, usually in the same paper. For meridians other than St Agnes, two bearings were generally used rather than one, and the chain of triangles was much longer.

When, for mapping purposes, the co-ordinates were plotted or projected on a plane rectangular grid, its grid lines were not exactly north/south, not exactly east/west, and not exactly to scale (except on the meridian of the origin).⁷

The horizontals of the grid represented great circles, not parallels of latitude, and the verticals did not exactly represent meridians (except on the origin). The 'rectangular' grids for the various meridians were at a slight angle to each other, according to the convergence of the meridians, as shown in *figure 1*.

⁶ The triangulation in figure 3 is from Royal Society 1797 at Tab XI, and angles and distances on pages 446-447 and 461 of the same paper were used by the writer to verify the calculation of co-ordinates listed in Royal Society 1800 page 635 and Art XXV (and used in table 3 below).

⁷ Importantly, the plane triangles used to calculate co-ordinates were not on the same plane, but formed the edges of part of an irregular polyhedron having vertices on the spheroid.

As well as for primary stations, tables of co-ordinates were listed for secondary stations such as church spires, windmills, lighthouses and other landmarks (about a thousand in total). Immediately following each table of co-ordinates, the Ordnance provided a table of latitudes and longitudes (as table 2). There was no explanation of the conversion process. As discussed below, this required assumptions (not always stated) as to the lengths of a degree on and perpendicular to the meridian. For calculations of longitude, the difference between lengths of the arcs of the parallel and the great circle appears to have been treated as insignificant.⁸ But calculations of latitude needed to be adjusted.

Latitude adjustment of Ordnance co-ordinates

The latitude of a point Q on the meridian of the origin O may be calculated by dividing its north co-ordinate OQ by the assumed length of a degree on this meridian (and adding this to the latitude of the origin). However, the latitude L of points P and S, with the same north co-ordinate as Q, but offset from this meridian, is slightly less, as shown in *figure 4*.

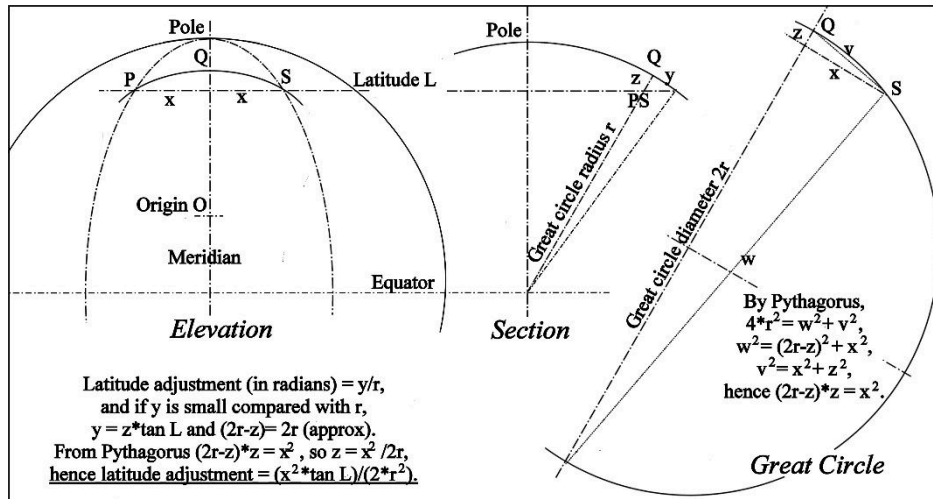


Figure 4: Diagram illustrating latitude adjustment of Ordnance co-ordinates

Conversion of Ordnance co-ordinates to Latitudes and Longitudes

Crucially, this conversion process required assumptions as to 'the figure of the earth'. In the early nineteenth century, there was nearly agreement that the shape of the earth could be represented by an oblate spheroid (ie a sphere squashed at the poles). However, the dimensions of this figure were still being investigated on the ground and debated in scientific journals.

For the conversion of co-ordinates to latitudes and longitudes, the 'figure of the earth' was used as a shorthand for the assumed lengths of a degree on the meridian and perpendicular to it, both figures having regard to latitude.

By making assumptions of the lengths of a degree, on and perpendicular to the meridian, and using a latitude adjustment as outlined above, a process for

⁸ For an arc of one degree at 60 deg N, arc parallel/arc great circle = $2 * \arcsin(x/r) / \arcsin(2x/r)$ and the difference between this arc on the great circle and on the parallel is 0.14 seconds.

converting co-ordinates into latitudes and longitudes may be re-constructed. Table 3 applies this to data published in 1800 for stations related to the meridian of St Agnes Beacon. This example was chosen because, in this case, the figures assumed for fathoms (of six feet) per degree were made clear by the Ordnance.

Meridian of St Agnes Beacon																
		Roy Soc 1800 p641				deg	min	secs	radians		fathoms per degree =					
		St Agnes Beacon Latitude N				50	18	27	0.8780		60845		on meridian as 1800 p 635			
		St Agnes Beacon Longitude E				-5	-11	-55.7	-0.0907		61182		on perpendicular ditto			
Station	Co-ordinates		Latitude N & Longitude E unadjusted						Latitude N adjusted					L&L reported		
	Roy Soc 1800													Roy Soc 1800		
	pp 643 & 653								adjustment					pp 644 & 655		
A	B	C	D	E	F	G			H	J	K			L		
		feet	origin	add	calc'd	deg	min	secs	deduct	calc'd	deg	min	secs	deg	min	secs
			radians	radians	radians				secs lat	radians						
Cadon Barrow	'N'	126702	0.8780	0.0061	0.8841	50	39	16.4	4.1	0.8841	50	39	12.4	50	39	12.1
	'E'	119364	-0.0907	0.0090	-0.0818	-4	-41	-9.3						-4	-41	-9.2
Trevose Head	'N'	88250	0.8780	0.0042	0.8823	50	32	57.2	0.5	0.8822	50	32	56.7	50	32	56.5
	'E'	42858	-0.0907	0.0032	-0.0875	-5	0	-54.2						-5	0	-54.2
Karnbonellis	'N'	-45379	0.8780	-0.0022	0.8759	50	10	59.5	0.0	0.8759	50	10	59.5	50	10	59.4
	'E'	-2742	-0.0907	-0.0002	-0.0909	-5	-12	-37.7						-5	-12	-37.7
St Burian	'N'	-83807	0.8780	-0.0040	0.8740	50	4	40.6	2.5	0.8740	50	4	38.1	50	4	37.9
	'E'	-94832	-0.0907	-0.0070	-0.0978	-5	-36	-4.9						-5	-36	-4.9

Table 3: Calculation of Latitudes and Longitudes from St Agnes Beacon

Col. C shows Ordnance co-ordinates on the meridian ('N') and perpendicular to it ('E'). Col. D takes the Ordnance latitude and longitude of the origin from the table heading. Col. E converts col. C from feet into angular distances, using the conversion factors (fathoms/degree) shown in the heading. Radians on the meridian convert directly, and radians perpendicular to the meridian are scaled by the cosine of the adjusted latitude. Cols. F and G show latitudes and longitudes calculated as the sum of cols D and E. Col. H calculates the latitude adjustment, from the formula derived above, using latitude L from col. F, distance x from col. C, radius r from fathoms/degree on the perpendicular.

In cols. J and K, latitudes shown in cols. F and G are adjusted by deducting col. H. Col. L shows latitudes and longitudes calculated and reported by the Ordnance.

After (and only after) making the latitude adjustment, the calculations shown in table 3 demonstrate a good match with the Ordnance results. With a very few exceptions, similar matches were achieved for stations measured from the meridians of Buttern Hill, Black Down, Dunnose and Greenwich, as reported in and before 1800. This suggests that the Ordnance process for converting co-ordinates into latitudes and longitudes made use of this or a similar latitude adjustment⁹ (and also disregarded the small difference between calculations made on a sphere or a spheroid).

⁹ In contemporary papers, this seems to be discussed only once, at Royal Society 1795, 520.

However, analysis of latitudes and longitudes published in 1811¹⁰ seems to reveal inconsistent assumptions. In that lengthy report, the assumed conversion factor adopted for fathoms to degrees on the meridian is not stated, although its preface refers to the factor of 60823 fathoms/degree calculated in 1803 between Dunnose and Burleigh Moor. As calculations (shown in table 4) based on 60823 fathoms per degree do not provide a very good match, these were repeated using the Admiralty convention of 6080 feet per nautical mile (60800 fathoms per degree). For stations measured from the meridian of Delamere this (in table 4) provides a remarkably good match. Moreover, this conversion factor of 60800 was found also to provide good matches for stations measured in 1811 from the meridians of Burleigh Moor, Clifton Beacon and Moel Rhyddlad.

Meridian of Delamere Forest						deg min secs			radians			fathoms per degree						
			Delamere Latitude N			53	13	20.8	0.9289				perpendicular to					
			Delamere Longitude E			-2	-40	-30.8	-0.0467				the meridian =			61182		
Co-ordinates			Common Data		Calculated Latitude& Longitude				L&L reported			Calculated Latitude& Longitude						
1811 vol III					including latitude adjustment				1811 vol III			including latitude adjustment						
pp 374-381					60823 fathoms/deg on merid'n				pp 374-381			60800 fathoms/deg on merid'n						
A	B	C	D	H	E	J	K		L			E	J	K				
			feet	origin	deduct	add	calc'd	deg min	secs	deg	min	secs	add	calc'd	deg min	secs		
				radians	secs lat	radians	radians						radians	radians				
East Lomond	'N'	1103315	0.9289	4.1	0.0528	0.9817	56	14	40.5	56	14	44	0.0528	0.9817	56	14	44.6	
(Firth of Forth)	'E'	-108892	-0.0467		-0.0093	-0.0560	-3	-12	-32.7	-3	-12	-33						
The Cheviot	'N'	824364	0.9289	4.3	0.0394	0.9683	55	28	48.6	55	28	52	0.0394	0.9683	55	28	51.7	
	'E'	112071	-0.0467		0.0094	-0.0373	-2	-8	-11.3	-2	-8	-12						
Holme Moss	'N'	116690	0.9289	9.6	0.0056	0.9344	53	32	22.3	53	32	23	0.0056	0.9344	53	32	22.7	
	'E'	174440	-0.0467		0.0140	-0.0327	-1	-52	-32.0	-1	-52	-32						
Flatholme	'N'	-673438	0.9289	2.9	-0.0322	0.8967	51	22	34.7	51	22	33	-0.0322	0.8967	51	22	32.2	
Lighthouse	'E'	-98908	-0.0467		-0.0075	-0.0542	-3	-6	-24.8	-3	-6	-25						

Table 4: Calculation of Latitudes and Longitudes from Delamere Forest

Why in 1811 was a shorter degree adopted in the north than in the south?

So why was it that the tables of latitudes and longitudes published for the Ordnance were in 1811 based on lengths of a degree on the meridian that were assumed to be less where measured from origins in the north than where measured from origins in the south?

Given the general agreement between geodesists of different countries, and even between the Admiralty and the Ordnance, that the figure of the earth approximated to an oblate spheroid, the length of a degree in feet was expected to increase from the equator to the pole. Indeed, the Ordnance made its calculations from origins in southern England using lengths of a degree on the meridian calculated on this basis.¹¹

¹⁰ *An Alphabetical List of the Latitudes and Longitudes of the principal stations, together with several Church Steeples, Lighthouses, and other remarkable objects*, 374-382, vol III, Wm Faden, 1811.

¹¹ Royal Soc 1795, 537: 60851 fathoms at latitude 50° 41', 60859 at 51° 5' and 60868 at 51° 28' 40".

Thus it was surprising that Major William Mudge, as he then was, had the confidence in 1803, having deduced that the length of a degree on the meridian was greater between Dunnose and Arbury Hill than it was on the more northerly arc between Arbury Hill and Clifton Beacon, to declare himself 'perfectly convinced of the general accuracy of the whole'.¹² He had perhaps been carried away by the potential of Ramsden's zenith sector, rebuilt in 1801, to improve the accuracy of stellar observations, and thereby the measurement of degrees.

However, Mudge's findings were inconsistent with the figure of the earth well documented by French geodesists and were fiercely criticised in a paper by Don Rodriguez, a Spaniard of their school, published by the Royal Society in 1812.¹³ Although these criticisms were soon disputed, it was also soon accepted that Mudge's stellar observations had been distorted by the effect on the sector's plumb bob of underlying differences in the density of the earth.

Rob Wheeler's article¹⁴ makes the significant point that the desire of the Royal Society to determine more accurately the shape of the earth infused the early operations of the trigonometrical survey. He suggests that Mudge perhaps hid his assumptions to avoid the ire of Fellows who held divergent views about the shape of the earth. In 1803, hastening to improve on the work of the French geodesists, Mudge may have failed to seek a second opinion on his own work.

By 1811 Mudge was probably nervous of the storm Don Rodriguez was about to unleash. The perfectionist Thomas Colby had largely taken charge of day to day operations, and it is not impossible that the Ordnance computers, originally trained by Isaac Dalby and impatient of waiting for Colby's instructions, pressed ahead by adopting the familiar figure of 6080 feet per minute of latitude.

Mudge and Colby must have been mortified by the Royal Society's willingness to publish the challenge to their cherished trigonometrical survey. The atmosphere could hardly have been improved by the Society's appointment in 1816 of Captain Henry Kater FRS to conduct gravitational experiments over the length of the Ordnance arc, or by Kater's constructive criticism of their assumed lengths of degrees of latitude.¹⁵ Apparently no-one suspected that ten years later the Ordnance would also have to reconsider the lengths of degrees of longitude.

Nevertheless, the three volume *Account of the Trigonometrical Survey* completed in 1811 was very open and informative, and it published useful latitudes and longitudes for perhaps a thousand stations over most of England and Wales and southern Scotland. Although Colby was unwilling until 1846 to

¹² Royal Soc 1803, 488-89: mid-point figures of 60864 fathoms at 51° 35' 18.2" (Dunnose to Arbury Hill) and 60766 fathoms at mid-point 52° 50' 29.8" (Arbury Hill to Clifton Beacon).

¹³ Don Joseph Rodriguez, *Observations on the measurement of three degrees of the meridian conducted in England by Lieut Col William Mudge*, Phil. Trans. Roy. Soc. Lond., vol. 102, 321-521, 1812.

¹⁴ RC Wheeler, 'William Mudge and the General Map of England', *Sheetlines* 97, 13-19.

¹⁵ Capt Henry Kater FRS, *An account of experiments for determining the variation in the length of the pendulum vibrating seconds, at the principal stations of the Trigonometrical Survey of Great Britain.*, Phil. Trans. Roy. Soc. Lond., vol 109, 337-508, 1819.

show latitudes and longitudes on Ordnance maps (and at first only on six inch maps),¹⁶ they were readily adopted by other topographical surveyors and by hydrographers.

Topographical Surveys

For the county mapmakers of the early nineteenth century, capable in topographical survey but only in local triangulation, even a few Ordnance positions must have been invaluable in providing reliable baselines. Christopher Greenwood lost no time in taking advantage of the Ordnance figures. Of his numerous maps in the British Library catalogue, that of the county of Yorkshire in 1817 was said to be ‘made on the basis of triangles in the county determined by Lt. Col. W Mudge and Capt. Thos. Colby’.¹⁷

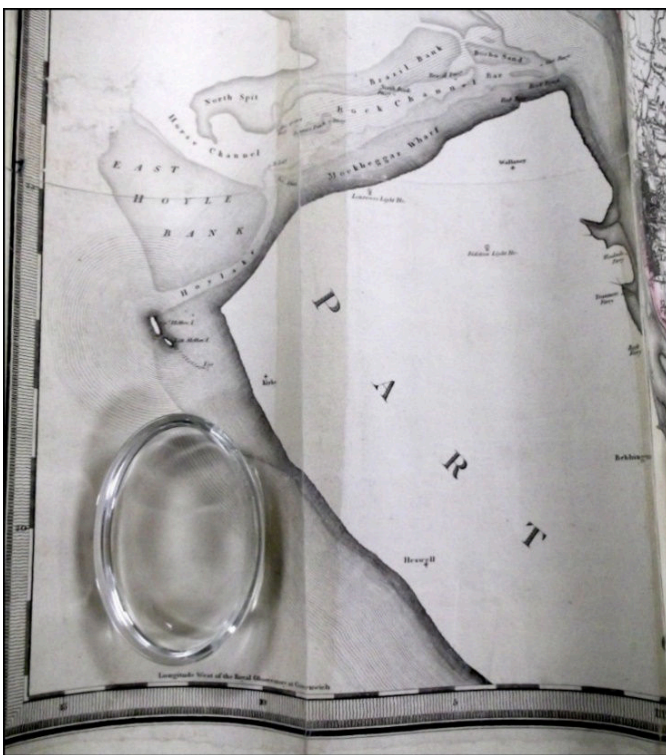


Fig 5: extract from Greenwood's map of Lancashire¹⁸

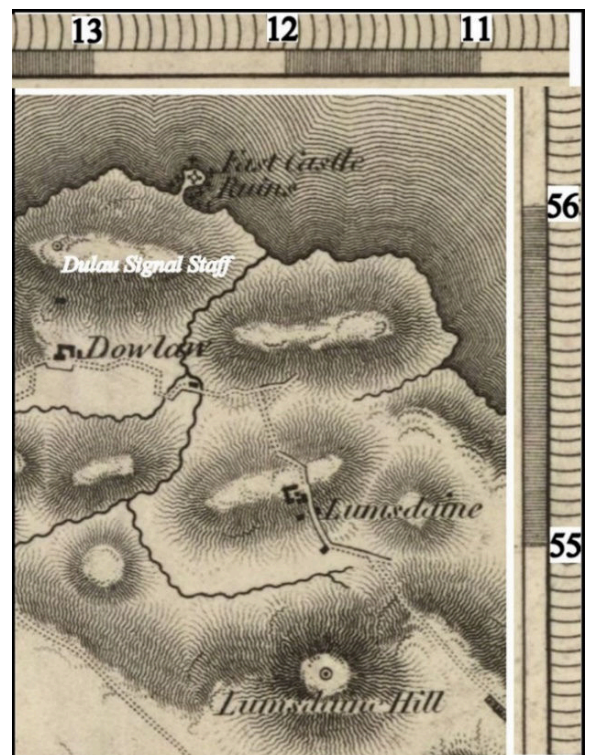


Fig 6: extract from Greenwood's map of Berwickshire¹⁹

Greenwood's county maps in the National Archives (only two of them) show marginal latitude and longitude graticules.¹⁸ The absence of topographical detail

¹⁶ Roger Hellyer & Richard Oliver, *The First Ordnance Survey Map*, 78, Charles Close Society, 2015.

¹⁷ John Hurst and C Greenwood, *Map of the County of York etc*, The British Library, C.23.c.20, 1817.

¹⁸ *Maps in 3 sheets of the County Palatine of Lancaster, taken from a survey made by C Greenwood, Wakefield. Scale: 1 inch to 1 mile*, The National Archives (TNA), MR 1/972, 1818; *County of Southampton: map from an actual survey made in 1825 and 1826 by C and J Greenwood and NL Kentish*, TNA, F 25/3, 1826.

for the Wirral, on the map of Lancashire, reveals three Ordnance stations from the list published in 1811 (Leasowes LH, Bidston LH and Bebbington Spire), at the corresponding latitudes and longitudes (*figure 5*).

In Scotland, where the Ordnance list of latitudes of longitudes extended only slightly north of the Firth of Forth, county maps of southern Scotland by Greenwood and colleagues were ‘made on the basis of the Trigonometrical Survey of Scotland’.¹⁹ These maps also show marginal latitude and longitude graticules, and several trigonometrical stations on each sheet are shown at the positions tabulated by the Ordnance in 1811. In Berwickshire, Dulau Signal Staff and Lumsdane Hill stations are marked with the appropriate symbol (*figure 6*).

Hydrographic Surveys

As Admiralty charts normally adopted the Mercator projection, the Ordnance from 1795 onwards calculated and reported latitudes and longitudes intended for the Admiralty, and this information was used to update Admiralty charts for much of the south coast.²⁰

The Ordnance papers completed by William Faden in 1811 were known by working hydrographers as ‘the three volumes’, and were drawn upon by them in a variety of ways. For example, as early as 1813, George Thomas pin-pointed fourteen of the secondary stations listed in 1811 on his chart of the Mersey.²¹ By observing two angles between these stations from his brig using a sextant, he could plot his position at sea using a ‘station pointer’, as illustrated in *figure 7*.

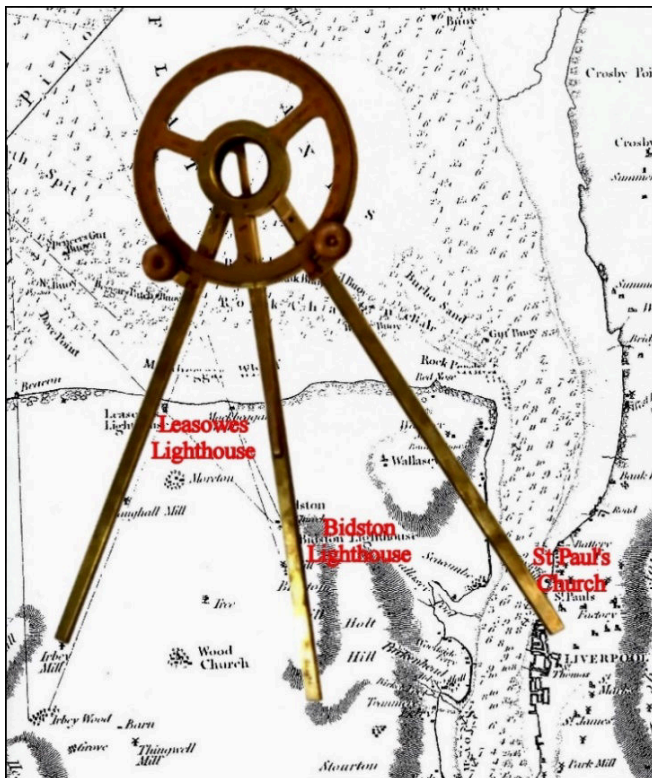


Fig 7: extract of Mersey chart ²¹ annotated to show use of station pointer

¹⁹ Thos Sharp, C Greenwood & Wm Fowler, *Maps of the Counties of Haddington 1825, Berwick 1826, Edinburgh 1828 and Fife & Kinross 1828*, National Library of Scotland, <http://maps.nls.uk>.

²⁰ David L Walker and Adrian Webb, ‘Some collaboration between the Ordnance Survey and the Hydrographic Office in the Nineteenth Century’, *Sheetlines* 102, 7.

²¹ George Thomas, Master RN, *Survey of the Harbour of Liverpool*, UKHO, 684a Dg, 1815.

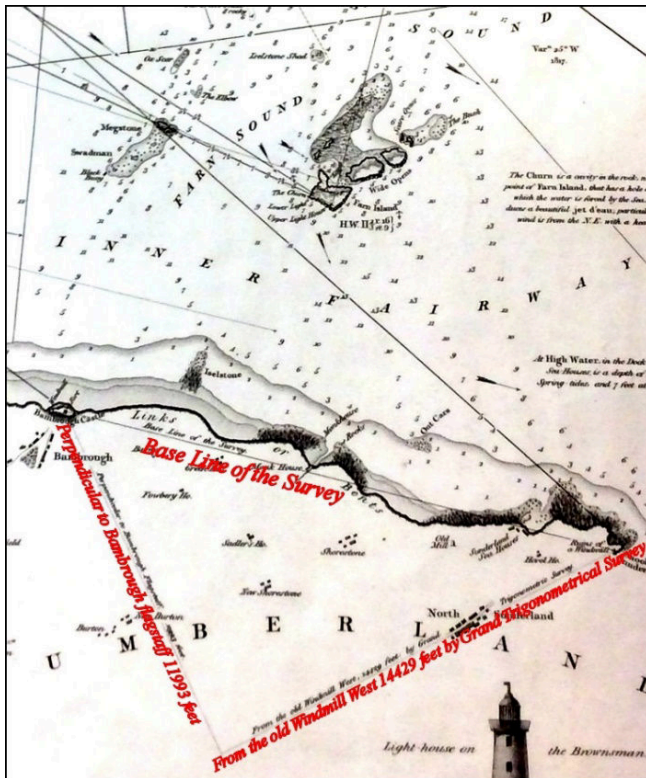


Fig 8: extract of Farne chart ²²
annotated to show Johnson's baseline

their distance between Beadnel Spire and Sunderland Mill, as deduced from two sides and the included angle, as a scale for my triangulation, deeming such to be, although only intersections, much more accurate than any baseline which I could measure with my present means ...

Nevertheless, Slater enjoyed a productive relationship with officers of the Ordnance that saw the survey of the east coast of Scotland almost completed in the 1830s. Over the same period, triangles extended by the Ordnance from its initial triangulation provided baselines for Admiralty surveys of Orkney and Shetland and from the Solway to the Clyde.²⁴

New assumptions for the figure of the earth

Very significantly, Lt Hastings Murphy of the Ordnance in 1830 provided the Admiralty Hydrographer with corrected positions for five stations on the Northumberland coast and three stations near the Scottish border.²⁵ The latter differed by about 10 seconds (south) in latitude and nearly 30 seconds (west) in

Lt EJ Johnson in 1817 used Ordnance co-ordinates listed in 1811 to define the baseline for his freelance survey of the Farne Islands, as in figure 8.²²

In 1824 Thomas made an impressive survey of the Gabbard shoals (off Suffolk) which he positioned by reference to the latitude and longitude of Orford Church as published in the Royal Society paper of 1803.²³

In 1828 Lt Michael Slater was despatched by the Admiralty to the Northumberland coast. Like George Thomas, he was an accomplished surveyor who occasionally corrected the Ordnance, and liked to make his own triangulation of coastal stations between two positions provided by the Ordnance. Slightly grudgingly, he wrote that

most of the definite objects used by them have long since been removed. I have however assumed

²² Lt EJ Johnson, *Chart of the coast of Northumberland [etc] and Farn Islands*, UKHO, E268 Df, 1819.

²³ Mr George Thomas RN, *A survey of part of the coast of Suffolk [and Gabbard shoals]*, UK Hydrographic Office (UKHO), MP 98, 1824; Royal Society 1803, 506.

²⁴ David L Walker and Adrian Webb, 'Some collaboration [etc]', *Sheetlines* 102, 11-14.

²⁵ Lt Hastings Murphy RE, UKHO, MP 98, 16 Jan 1830 & 2 April 1830.

longitude from those listed in 1811. It seems that these arose not from new surveys, but from a correction of Mudge's over-estimate of the length of a degree perpendicular to the meridian that is identified in the 'official' history.²⁶

It is at first unclear whether Murphy's revised assumptions for the lengths of degrees on and perpendicular to the meridian came from the appendix to William Lambton's paper published in 1818²⁷ or advice from Professor Airy in advance of the paper he published later in 1830.²⁸ This is clarified by a contemporary letter from Murphy²⁹ apologising for an error in positions on the Lincolnshire coast he correctly computed from the local meridian 'on the spheroid of Lambton & Delambre' but displaced by an error in the position of Clifton Beacon.

Lambton's figure of the earth, apparently ignored for years, was easy for others to use as he tabulated his conclusions as lengths of degrees calculated on and perpendicular to the meridian at three-degree intervals of latitude. It was also adopted by Lt Denham RN in April 1830 in making a comparison for four stations in Pembrokeshire between latitudes and longitudes published in 1811 and figures 'as recomputed on the spheroid of Delambre & Lambton'.³⁰

In 1834 Murphy provided corrected positions for additional stations in eastern Scotland.³¹ *Figure 9* shows how these latitudes and longitudes, recalculated by Murphy from the initial triangulation, are impressively close (for the only two stations in both surveys) to those reported in the twentieth century retriangulation.³² At Kellie Law both differences are less than one second and at Brimmond less than two seconds.

Murphy 1834	Latitude	Longitude	20c Retriangulation
Largo Law (Trig. S)	56 - 14 - 16.8	2 - 55 - 27.7	
Kellie Law Do	56 - 14 - 53.1	2 - 46 - 42.9	Kellie Law (in Fife): Latitude 56 deg 14' 53.744" Longitude 2 deg 46' 43.72822"
East Lomond Do	56 - 14 - 30.9	3 - 13 - 10.2	
Red Head Do	56 - 36 - 55.3	2 - 29 - 24.7	
Bell Rock L.H. Do	56 - 26 - 03.4	2 - 23 - 6.1	
Brimmond Hill Do	56 - 10 - 19.8	2 - 14 - 11.6	Brimmond (near Aberdeen): Latitude 57 deg 10' 21.0753" [56 deg appears to be mistaken] Longitude 2 deg 14' 13.3158"
* Gail Spiv. O. Inters	56 - 15 - 45	2 - 37 - 26	

*Figure 9:
Positions
corrected in
1834 compared
with C20
retriangulation*

²⁶ WA Seymour et al, *A History of the Ordnance Survey*, 39, Dawson, 1980.

²⁷ William Lambton, *An abstract of the results deduced from the measurement of an arc on the Meridian [etc]*, Phil. Trans. Roy. Soc. Lond., Vol 108, 517, 1818.

²⁸ GB Airy, *The Figure of the Earth*, Encyclopaedia Metropolitana, 1830, reprinted in vol V, 165-239, 1848.

²⁹ Lt Hastings Murphy RE to Capt Francis Beaufort RN, UKHO, LP1857 M, 557, 4 Jan 1830.

³⁰ Lt HW Denham RN, *Differences in Longitudes, Caernarvon Bay*, UKHO, MP 98, 3 April 1830.

³¹ Lt MA Slater RN to Capt Beaufort RN, *Latitudes & longitudes from Lt Murphy*, UKHO, SL 6a, 6 Dec 1834.

³² Ordnance Survey, *The History of the Retriangulation of Great Britain 1935-1962*, Appendix 10, Figure 3, 1967.

The need for latitude corrections is understandable by reference to the problems arising from the measurement of the arc between Dunnose and Clifton Beacon. The longitude corrections, which were actually greater, appear to correct Mudge's over-estimate, mentioned above, of the length of a degree perpendicular to the meridian.

Murphy's letter of 4 January 1830 implies that Captain Alexander Robe was at least aware of his use of Lambton's figure of the earth to correct latitudes and longitudes supplied to the Admiralty. However, the corrected latitudes and longitudes (including figures not reported here) that he and others supplied to the Admiralty from 1834 onwards appear to be based on Airy's figure of the earth.

Conclusions

This article illustrates the method used by the Ordnance to calculate the co-ordinates of stations observed in its initial triangulation of Great Britain. The illustration remedies the over-simplified statement in the 'official' history that computations were made 'on the assumption that the Earth's surface was plane'.³³ Worked examples of a 'latitude adjustment' reconstruct a feasible process for the conversion of these co-ordinates into latitudes and longitudes.³⁴

A reminder is included that, although the initial Ordnance triangulation provided a useful basis for the mapping of England and Wales, it was less successful in measuring the shape of the earth. This perhaps explains why the Ordnance for decades lost its confidence in geodesy, failed to engage in outside discussion and delayed the disclosure of its work.

Although the comprehensive tables of latitudes and longitudes provided in its published papers were not used by the Ordnance itself, these were seized upon by some independent mapmakers. The provision of corrected figures to the Admiralty Hydrographer unexpectedly reveals the unofficial adoption of Lambton's figure of the earth in 1830. Further advice from the Ordnance, based on extensions of the initial triangulation and apparently using Airy's figure of the earth, provided a sound basis for the hydrographic survey of the Scottish coast.

A comparison of latitudes and longitudes supplied by the Ordnance in the 1830s with the 20th century re-triangulation suggests that the initial triangulation may be regarded as vindicated by its projection on Airy's figure of the earth.

³³ WA Seymour et al, *A History of the Ordnance Survey*, 39, Dawson, 1980.

³⁴ Although this conversion process does not appear to be explained in the Royal Society papers, or Faden 1811, AR Clarke's method for converting rectangular spheroidal co-ordinates to latitude and longitude is explained in AJ Wolff, *Mathematical Basis of the Ordnance Maps of the United Kingdom*, Southampton, 1919, 11-12. As a first approximation 'which generally suffices' this results in the same latitude adjustment as that derived above.

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Appendix

The following papers published on behalf of the Ordnance are referred to above by the years of publication. The Royal Society papers can be down-loaded from its website and were reprinted (with different pagination) by Faden as vol. I (in 1799) and vol. II (in 1801) of *An account of the Trigonometrical Survey etc'*.

Lt Col Edward Williams, Capt William Mudge of the Royal Artillery and Mr Isaac Dalby, *An Account of the Trigonometrical Survey carried on in the years 1791, 1792, 1793, and 1794, by Order of His Grace the Duke of Richmond, Late Master General of the Ordnance*. Phil. Trans. Roy. Soc. Lond., vol 85, 414-591, 1795.

Col Edward Williams, Capt William Mudge and Mr Isaac Dalby, *An Account of the Trigonometrical Survey carried on in the years 1795 and 1796 etc*, Phil. Trans. Roy. Soc. Lond., vol 87, 432 -541, 1797.

Capt William Mudge, of the Royal Artillery, *An Account of the Trigonometrical Survey carried on in the years 1797, 1798, and 1799 etc*, Phil. Trans. Roy. Soc. Lond., vol 90, 539-728, 1800.

Major William Mudge, of the Royal Artillery, FRS., *An account of the measurement of an arc of the Meridian, extending from Dunnose, in the Isle of Wight, latitude 50° 37' 8", to Clifton, in Yorkshire, latitude 53° 27' 31" in the course of the operations carried on for the trigonometrical survey of England, in the years 1800, 1801, and 1802*, Phil. Trans. Roy. Soc. Lond., vol 93, 383-508, 1803.

Lt Col William Mudge RA., FRS and Capt Thomas Colby RE, *Account of the Trigonometrical Survey etc carried on in the years 1800 [to] 1809*, Vol.III, sold by W Faden, Geographer to His Majesty, 1811. This can be found in a Google search.